PERFROMANCE EVALUATION OF TWO SIGNATURES' CODES LINEAR DECORRELATOR DETECTOR FOR CDMA SYSTEMS IN NAKAGAMI FADING CHANNEL

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ABSTRACT

In linear time invariant channel model, the systems that are designed based on CDMA are suffering from multiple access interference (MAI) and noise problems. In time varying channel model, the CDMA systems suffer from complex time varying channel's gains plus MAI and noise. A lot of CDMA detectors are designed to overcome the MAI problem. But these detectors have complex structures and their performances are affected by the presence of time varying channel's gains. This paper gives proposal for new linear CDMA detector that has the same MAI cancellation capability as CDMA decorrelator detector but with simpler structure. The operation of the new detector is based on the symmetry property of the cross-correlation matrix of CDMA signatures' codes. The structure complexity of this new detector is as simple as matched filter detector structure. The performance of this new detector is evaluated in the presence of Nakagami time varying channel's complex gains. Closed forms of probability of error for the proposed detector and decorrelator detector are represented for flat and multipath Nakagami fading models. Channel phase estimation error effect is included in the probability of error formulas. Different channel estimators are used with the proposed detector to overcome the fading phenomenon.

KEYWORDS

Code division multiple access (CDMA)–multiple access interference (MAI)–matched filter–decorrelator detector–signature codes correlation matrix–Channel flat fading–Multipath fading–Rayleigh fading–Nakagami fading–Channel estimation phase error.

1. INTRODUCTION

Linear CDMA detectors are widely used in CDMA systems' design because the complexity of these detectors is linear with the number of system's users [2]. Matched filter, Decorrelator, and MMSE adaptive filter are examples of these linear detectors. CDMA system is interference limited system where the multiple access interference (MAI) signals from system's users that affect the desired user signal is the most influential factor on the performance of this desired user signal [3]. Matched filter detector is the simplest CDMA detector. It is the optimum receiver of a known signal in AWGN environment [1]. But in CDMA system, the matched filter is not the optimum receive because the power of system's MAI signals is very high at the output of the matched filter [4]. On the other hand, the decorrelator detector is the linear CDMA detector that can completely cancel all MAI signals at the output of the detector [2]. But the decorrelator detector enhances the Gaussian noise power at the detector output. Also, the structure of the decorrelator detector is quiet complex where the detector should know all the signatures' codes of all system users in order to form the detector matrix that represents the inverse of the cross-correlation matrix among the system users' signatures' codes [5]. So the complexity of the decorrelator detector is greater than the complexity of the matched filter detector. The MMSE detector is an adaptive algorithm detector that compromises between the matched filter detector and the decorrelator detector [6]. The MMSE detector minimizes the MAI signals' powers and the noise power jointly at the output of the detector. The MMSE detector needs to know the desired user signature code only. So the structure of the MMSE detector is simpler than the structure of decorrelator detector. But the MMSE detector is still complex with respect to matched filter detector. MMSE detector needs a training sequence in the initiation of the communication link to adjust the MMSE adaptive filter taps. During communication course, the adaptive algorithm is working in decision directed mode to minimize the MMSE between the income signal and the detector output.

If the linear CDMA detectors are used in time varying channel, the performance of these detectors will be got worst. So, channel estimation should be used to estimate the time varying complex channel's gains. The receiver uses this channel estimation to compensate the fading effect of the channel. But the channel estimation increases the complexity of the receiver which is already having a complex structure to eliminate or reduce the MAI problem. Also the channel estimation process is affected by the system MAI signal where the mean square error between the actual channel fading gains and the estimated gains may be high. This estimation error affects the channel fading compensation process on the receiver. A lot of trials have been done to eliminate the problems of channel fading and system MAI. But these trials give complex receivers structures. Also these proposed receivers have degraded performance when the channel fading gets fast where the conventional channel estimation algorithms failed to track the fast variation in channel gains.

Here, a new linear CDMA detector is proposed to approach the MAI cancellation performance of the decorrelator detector but with simpler structure as matched filter detector structure. This detector is based on a mathematical observation relating to the symmetry property of the cross-correlation matrix among the CDMA system users' signature codes [7]. This new proposed detector with simpler structure may help in designing a simple receiver that can solve the problems of channel fading and MAI altogether. The proposed receiver uses a pilot signal to estimate the time varying channel's complex gains. This new proposed receiver with simpler structure may help in increasing CDMA system capacity by allowing more number of system's users to share the same CDMA system's resources.

The reminder of this paper is organized as follows. In section (2), the mathematical system model is represented. This model helped in understanding the system behavior and the problems that are faced. Section (3) shows the main idea of the new proposed detector. The system structure of the new proposed detector is also represented in this section. The probability of error for the new proposed detector is represented in section (4) for linear time invariant channel. Also in this section, the average probability of error for the proposed detector and the decorrelator detector are calculated for the cases of flat and multipath Nakagami channel models. Also they are compared with the average probability of error for BPSK system at the same conditions. In section (5), the simulation results of the proposed detector are shown. The simulations results include comparison among the proposed detector, the decorrelator detector and the matched filter detector. The comparisons are done for the cases of Nakagami flat fading with m=1 & 2 and Doppler frequency of 1KHz & 100 KHz. Different channel estimation algorithms are used in this comparisons. Finally the conclusions and future works are contained in section (6).

2. SYSTEM MODEL

Multiuser CDMA detectors commonly have a front end whose objective is to obtain a discrete time process from the received continuous time waveform y(t).

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma\ n(t), \qquad t \in [0,T] \qquad (1)$$

The notation introduced in Eq.(1) is defined as followed. **T** is the inverse of the data rate. $\mathbf{s_k}(t)$ is the deterministic signature waveform assigned to the k^{th} user, normalized so as to have unit energy.

$$\|s_k\|^2 = \int_0^T s_k^2(t).dt = 1 \qquad (2)$$

The signature waveform are assumed to be zero outside the interval [0, T], and therefore, there is no intersymbol interference. $\mathbf{A_k}$ is the received time varying Rayleigh random gain of the k^{th} user's signal. E[A_k^2] is referred to as the energy of the k^{th} user. $b_k \in [-1,1]$ is the bit transmitted by the k^{th} user. $n(t)$ is white Gaussian noise with unit power spectral density. It models thermal noise plus other noise source unrelated to the transmitted signal. According to Eq.(1) the noise power in a frequency band B is $2\sigma^2$B.

Continuous to discrete time conversion can be realized by conventional sampling, or more generally, by correlation of y(t) with deterministic signals [2]. Two types of deterministic signals are of principal interest; the signature waveform and orthonormal signals [1].

One way of converting the received waveform into a discrete time process is to pass it through a bank of matched filters. Each filter is matched to the signature waveform of a different user. In the synchronous case, the output of the bank of matched filter is shown in Eq.(3).

$$y_1 = \int_0^T y(t).s_1(t).dt$$

$$y_2 = \int_0^T y(t).s_2(t).dt \qquad (3)$$

$$\vdots$$

$$y_K = \int_0^T y(t).s_K(t).dt$$

where y(t) is represent by Eq.(1). The output of the k^{th} matched filter can be expressed as in Eq.(4).

$$y_k = A_k b_k + \sum_{j\neq k} A_j b_j \rho_{jk} + n_k \qquad (4)$$

where:

$$\rho_{jk} = \langle s_j(t), s_k(t)\rangle = \int_0^T s_j(t) s_k(t).dt \qquad (5)$$

$$n_k = \sigma \int_0^T n(t).s_k(t).dt \qquad (6)$$

It is noted that by Cauchy- Schwarz inequality and Eq. (2), the absolute value of the correlation coefficient is give in Eq.(7).

$$|\rho_{jk}| = |\langle s_j(t), s_k(t)\rangle| \le \|s_j\|\|s_k\| \qquad (7)$$

n_k is a Gaussian random variable with zero mean and variance equal to σ^2. It is convenient to express Eq.(4) in vector form:

$$\mathbf{y} = \mathbf{RAb} + \mathbf{n} \qquad (8)$$

where:

- **R**= {ρ_{jk}} = {<s_j(t),s_k(t)>} is the normalized cross-correlation matrix.
- $\mathbf{y} = [\mathbf{y_1}, \mathbf{y_2}, \ldots, \mathbf{y_K}]^\mathbf{T}$
- $\mathbf{b} = [\mathbf{b_1}, \mathbf{b_2}, \ldots, \mathbf{b_K}]^\mathbf{T}$
- $\mathbf{A} = \mathbf{diag}[\mathbf{A_1}, \mathbf{A_2}, \ldots, \mathbf{A_K}]^\mathbf{T}$
- **n** is a zero mean Gaussian random vector with covariance matrix equal to:

$$E[\mathbf{nn}^T] = \sigma^2\mathbf{R} \qquad (9)$$

No information related to demodulation is lost by the bank of matched filters; in other words, y(t) can be replaced by **"y"** which is the ***sufficient statistic*** for the detection of users' data without loss of optimality [7].

3. PROPOSED CDMA LINEAR DETECTOR

From the previous discussion on the linear multiuser CDMA detectors, it was found that as the capability of the detector to cancel the MAI is increased, the complexity of the detector is increased too. The simplest CDMA detector is the matched filter. But the matched filter can not cancel the multiple access interference signals as shown in Eq.(14).

$$\underbrace{y_k}_{Detector\ \ output} = \underbrace{A_k b_k}_{Desired\ \ signal} + \underbrace{\sum_{j\neq k} A_j b_j \rho_{jk}}_{MAI} + \underbrace{n_k}_{Noise} \qquad (14)$$

The detector that can cancel the MAI signals completely is the decorrelator detector. But the structure of this detector needs to know the entire signature codes of the system's users. The decorrelator detector has a matched filter for each user signature. It calculates the correlation matrix among these users' signature codes. Then it calculates the inverse of this correlation matrix. Finally it multiplies this correlation matrix inversion to the matched filters output vector. The decorrelator detector can cancel all MAI signals but it enhances the channel noise. Eq.(15) shows the operation of the decorrelator detector [8].

$$\mathbf{R}^{-1}\mathbf{y} = \mathbf{R}^{-1}\mathbf{RAb} = \mathbf{Ab} \qquad (15)$$

Here a new question may be appeared, is it possible to have a detector that can cancel all the MAI signals with a simpler structure than the Decorrelator detector?

The two signatures decorrelator detector is the detector that may answer the previous question. The receiver structure idea is base on the symmetry property of the signatures' codes correlation matrix [7]. The symmetry property of the correlation matrix can be represented by the following equation.

$$\rho_{ij} = \rho_{ik} \qquad For\ all\ \ 0 < i\ \&\ j\ \&\ k < K \qquad (16)$$

where ρ_{ij} is the correlation coefficient between user i and user j. Also, ρ_{ij} can be represented as the element at row i and column j in the signatures' codes correlation matrix **R**. The correlation matrix **R** can be represented as:

$$\mathbf{R} = \mathbf{S}^H.\mathbf{S} = \begin{bmatrix} s_1^H \\ s_2^H \\ \vdots \\ s_K^H \end{bmatrix} \begin{bmatrix} s_1 & s_2 & \cdots & s_K \end{bmatrix} \qquad (17)$$

The operation of the two signatures decorrelator is based on using two matched filters to eliminate the MAI signals based of the symmetry property of the signatures' codes correlation matrix. The first matched filter is the desired user matched filter that correlates the input received signal with the signature code of the desired user. The second matched filter is the reference matched filter. This matched filter is matched with a reference signature code that is not used by any user in the system. This code is common in all receivers that use the working system. Eqs.(18-19) represent the output of the desired user (user k) matched filter and the reference matched filter respectively.

$$y_k = A_k b_k + \sum_{j\neq k} A_j b_j \rho_{jk} + n_k \qquad (18)$$

$$y_r = A_k b_k \rho_{rk} + \sum_{j\neq k} A_j b_j \rho_{jr} + n_r \qquad (19)$$

where:

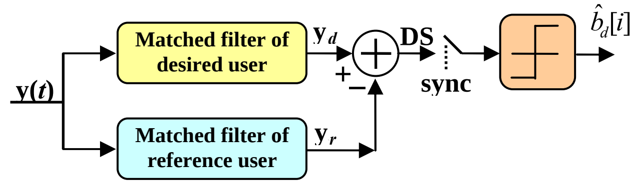

Fig.1. Proposed two signatures decorrelator detector structure.

$$\rho_{jk} = <s_j(t), s_k(t)> = \int_0^T s_j(t).s_k(t).dt \ ; \ n_k = \sigma\int_0^T n(t).s_k(t).dt$$

$$\rho_{jr} = <s_j(t), s_r(t)> = \int_0^T s_j(t).s_r(t).dt \ ; \ n_r = \sigma\int_0^T n(t).s_r(t).dt$$

and $\boldsymbol{s_k}(t)$ is the desired user signature code and $\boldsymbol{s_r}(t)$ is the reference signature code.

From the symmetry property of the correlation signatures' codes matrix **R**, it was found that:

$$\rho_{jk} = \rho_{jr} \qquad For\ all\ \ 0 < j\ \&\ k < K \qquad (20)$$

By subtracting Eq. (19) from Eq.(20), it was found that:

$$y_k - y_r = A_k b_k (1 - \rho_{rk}) + n_k + n_r \qquad (21)$$

So, the detector decision statistics can be taken to be equaled to the output in Eq.(21).

$$DS = A_k b_k (1 - \rho_{rk}) + n_k + n_r \qquad (22)$$

Now it is cleared that the proposed two signatures codes decorrelator detector has canceled all the MAI signals but on the cost of duplicating the back ground channel Gaussian noise.

The detector output which represents the estimate of the desired user data will be the sign of the decision statistics as in Eq.(23).

$$\hat{b}_k = \mathrm{sgn}(DS) = \mathrm{sgn}(A_k b_k (1 - \rho_{rk}) + n_k + n_r) \qquad (23)$$

Fig.1 shows the proposed two signature codes decorrelator detector structure. The advantages of the two signatures' codes decorrelator detector are:

1. Simple structures; the detector consists of two matched filters only instead of ***K*** matched filters as in the conventional decorrelator detector.
2. The detector does not need to know the number of system users nor the signatures' codes of them.
3. There is no need to neither calculate the inversion of the signatures' codes correlation matrix nor facing the problem of matrix singularity.

The disadvantage of two signatures' codes decorrelator detector is:

1. Noise enhancing; the noise power is increased by 3dB due to the duplication of noise component in decision statistics.

As it was shown, the proposed two signatures' codes Decorrelator detector is based on the idea of symmetry property of the correlation matrix of the CDMA signatures' codes. But if this condition is not satisfied for certain system codes, what will be the solution in this case?

The solution of the previous problem is not difficult. By using matrix algebra, the calculation of the reference signature code will be not difficult. For any working system, the correlation vector is calculated first between the desired user signature code and the other system signatures' codes as shown in Eq.(24).

$$\begin{bmatrix} s_1^H \\ s_2^H \\ \vdots \\ s_d^H \\ \vdots \\ s_K^H \end{bmatrix} .s_d = \begin{bmatrix} \rho_{d1} \\ \rho_{d2} \\ \vdots \\ 1 \\ \vdots \\ \rho_{dK} \end{bmatrix} \Rightarrow \Theta.s_d = \Delta \qquad (24)$$

where Θ is the systems' signature codes matrix, $\boldsymbol{s_d}$ is the desired user signature code vector, and Δ is the correlation between the desired user code and the other users' codes vector. The correlation vector Δ is used to calculate the reference signature code $\boldsymbol{s_r}$ after modifying the element of index $\boldsymbol{d}$ to be equaled to $\boldsymbol{\varepsilon}$ (small number) that represents the correlation between the desired user signature code and the reference signature code. Eq. (25) shows how the reference signature code can be calculated.

$$s_r = \Theta^{-1}.\Delta^{\backslash} \qquad where \quad \Delta^{\backslash} = \begin{bmatrix} \rho_{d1} \\ \rho_{d2} \\ \vdots \\ \varepsilon \\ \vdots \\ \rho_{dK} \end{bmatrix} \qquad (25)$$

In this case it is not necessary to have a common reference signature code for all systems receivers. On the other hand, each receiver may have its own reference signature code according to its desired user signature code as shown in Eqs.(24-25).

4. PROBABILITY OF ERROR CALCULATIONS

4.1 Linear Time Invariant Channel Model.

The probability of error calculation is always depending on the detector output before the decision rule. This detector output represents a random variable that is called a ***sufficient statistics***. In matched filter detector case, the probability of error of the desired user (user $\boldsymbol{k}$) that is a member in CDMA system is represents by Eq.(26) [1].

$$P_k^{mf}(\sigma) = \frac{1}{2} erfc\left(\frac{A_k}{\sqrt{2(\sigma^2 + \sum_{j \neq k} A_j^2 \rho_{jk}^2)}} \right) \qquad (26)$$

For the case of decorrelator detector, the desired user probability of error is represented in Eq.(27) [2].

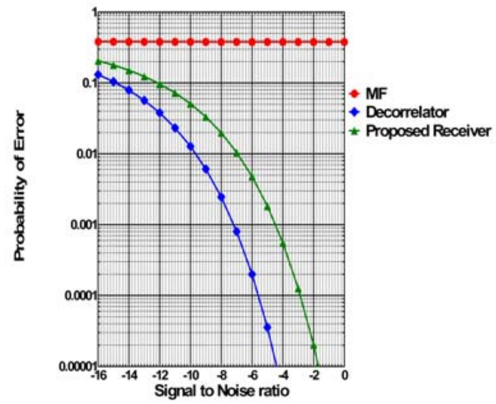

Fig.2. Probability of error for certain user in CDMA system using ML signature codes in linear time invariant channel and SIR=-40dB

$$P_k^d(\sigma) = \frac{1}{2} erfc\left(\frac{A_k}{\sigma\sqrt{2\mathbf{R}_{\mathbf{kk}}^{+}}} \right) = \frac{1}{2} erfc\left(\frac{A_k}{\sigma} \sqrt{\frac{\mathbf{1} - \mathbf{a_k^T R_k^{-1} a_k}}{2}} \right) \qquad (27)$$

where $\mathbf{a_k}$ is the k^{th} column of $\mathbf{R}$ without the diagonal element, and $\mathbf{R_k}$ is the (K-1)×(K-1) matrix that results by striking out the k^{th} row and column from $\mathbf{R}$. To obtain Eq.(27), the crosscorrelation matrix is assumed to be nonsingular.

The probability of error calculation in the case of the proposed detector is very easy. By referring to Eq.(22), the proposed detector decision statistic can be written as in Eq.(28)

$$DS_k = A_k b_k (1 - \rho_{rk}) + n_{kr} \qquad (28)$$

where n_{kr} is a zero mean Gaussian noise with $2\sigma^2$ variance. So from Eq.(28), the proposed detector probability of error can be represented as in Eq.(29).

$$P_k^{pd}(\sigma) = \frac{1}{2} erfc\left(\frac{A_k(1 - \rho_{rk})}{2\sigma} \right) \qquad (29)$$

Fig.2 shows the plot of probability of error verses signal to noise ratio in the desired user data at signal to interference ratio of -40 dB using Eqs.(26, 27, and 29) for matched filter detector, decorrelator detector and proposed detector respectively.

4.2 Nakagami Flat Fading Model

In Nakagami flat fading model, the desired user (user $\boldsymbol{k}$) has a complex Gaussian time varying channel gain $\boldsymbol{A_k}$. The amplitude of this gain represents Nakagami random variable and the phase of this gain represents uniform random variable. In any detector, the decision rule should be applied on real variable. So, the detector should calculate the real value of Eq.(28) before the **sign()** decision rule. Also, the detector should estimate the time varying complex gain to be able to compensate the effect of the channel fading. So, the new decision statistic is formed as the output of the dot product between the detector output and the estimated channel gain $\hat{A}_k$ as shown in Eq.(30).

$$DS^{m}{}_{k} = \hat{A}_k.(A_k b_k(1-\rho_{rk}) + n_{kr}) \quad (30)$$
$$= \left|\hat{A}_k\right|\left|A_k\right|\cos(\theta_e) b_k(1-\rho_{rk}) + \left|\hat{A}_k\right|\left|n_{kr}\right|\cos(\theta_{nk})$$

where $\boldsymbol{\theta_e}$ is the estimation phase error of user $\boldsymbol{k}$ and $\boldsymbol{\theta_{nk}}$ is the angle between the estimated complex channel gain vector and the complex Gaussian noise vector. The probability of error in the data that is estimated after applying the sign function on the modified decision statistic in Eq.(30) is represented by Eqs.(31-33).

$$P_k^{pd} = p[DS^{\backslash} > 0 \mid b_1 = -1]p[b_1 = -1] + p[DS^{\backslash} < 0 \mid b_k = +1]p[b_1 = +1] \quad (31)$$

$$P_k^{pd} = \frac{1}{2}p\left[-\left|\hat{A}_k\right|\left|A_k\right|\cos(\theta_e)b_k(1-\rho_{rk}) + \left|\hat{A}_k\right|\left|n_{kr}\right|\cos(\theta_{nk}) > 0 \mid b_1 = -1\right]$$
$$+\frac{1}{2}p\left[+\left|\hat{A}_k\right|\left|A_k\right|\cos(\theta_e)b_k(1-\rho_{rk}) + \left|\hat{A}_k\right|\left|n_{kr}\right|\cos(\theta_{nk}) < 0 \mid b_k = +1\right] \quad (32)$$

$$P_k^{pd} = \frac{1}{2}erfc\left(\frac{\left|A_k\right|(1-\rho_{rk})\cos(\theta_e)}{2\sigma\cos(\theta_{nk})}\right) \quad (33)$$

It should be noted that the term (**cos($\boldsymbol{\theta_{nk}}$)**) is a random variable where $\boldsymbol{\theta_{nk}}$ is uniform random variable that represents the angle difference between the noise vector and the estimated channel gain vector. In the worst case, this angle is equaled to zero. Also, the term (**cos($\boldsymbol{\theta_e}$)**) may be considered as a constant where good channel estimation algorithms can keep this factor constant and very small. So Eq.(33) can be written as:

$$P_k^{pd} = \frac{1}{2}erfc\left(\frac{\left|A_k\right|(1-\rho_{rk})\cos(\theta_e)}{2\sigma}\right) \quad (34)$$

Eq.(34) represents the probability of error of the proposed detector in Nakagami flat fading channel. But this probability of error is random variable because $\left|A_k\right|$ is Nakagami random variable. So by assuming that($x = \left|A_k\right|$), the average of Eq.(34) is calculated as:

$$E[P_k^{pd}] = \int_{-\infty}^{\infty} P_k(x).f_x(x).dx \quad (35)$$

$$f_x(x) = \frac{2}{\Gamma(m)}.\left(\frac{m}{\Omega}\right)^m .x^{2m-1}e^{\left(\frac{-mx^2}{\Omega}\right)} \quad (36)$$
$$\Omega = E[x^2]$$

where $f_x(x)$ is the probability density function of a Nakagami *m*-distribution random variable. From the properties of erfc(.) in [2], the following integration solution is represented.

$$\frac{1}{2}\int_0^{\infty} z^{2n-1}.e^{\left(\frac{-z^2}{2}\right)}.erfc\left(\frac{z}{\sqrt{2}\sigma}\right).dz$$
$$= \frac{(n-1)!}{2}(1-(\sigma^2+1)^{-1/2})^n.\sum_{k=0}^{n-1}2^{-k}\binom{n-1+k}{k}(1+(\sigma^2+1)^{-1/2})^k \quad (37)$$

So, by using Eq.(34) and Eq.(37), the solution of Eq.(35) is represented as:

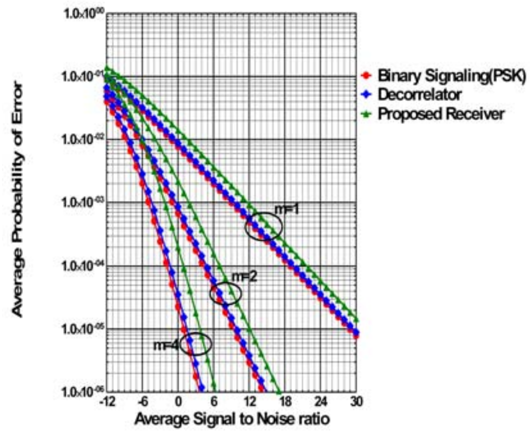

Fig.3. Average probability of error for certain user in CDMA system using ML signature codes in Nakagami flat fading channel with *m*=1, 2, &4 and SIR=-40dB

$$E[P_k^{pd}] = \frac{2^{-m}(m-1)!}{\Gamma(m)}(1-(v^2+1)^{-1/2})^m.\sum_{k=0}^{m-1}2^{-k}\binom{m-1+k}{k}(1+(v^2+1)^{-1/2})^k$$
$$v = \frac{\sqrt{2}\sigma}{(1-\rho_{rk})\cos(\theta_e)}\sqrt{\frac{2m}{\Omega}} \quad (38)$$

Eq.(38) represents the average probability of error of the two signature codes decorrelator (proposed detector) in flat fading Nakagami channel.

The same procedure is used to calculate the average probability of error for the decorrelator detector and binary signaling system (BPSK) in flat fading channel. Eqs.(39-40) shows the average probability of error of decorrelator detector and binary signaling system respectively.

$$E[P_k^{d}] = \frac{2^{-m}(m-1)!}{\Gamma(m)}(1-(v^2+1)^{-1/2})^m.\sum_{k=0}^{m-1}2^{-k}\binom{m-1+k}{k}(1+(v^2+1)^{-1/2})^k$$
$$v = \frac{\sigma}{\sqrt{\mathbf{1-a_k^T R_k^{-1} a_k}}\cos(\theta_e)}\sqrt{\frac{2m}{\Omega}} \quad (39)$$

$$E[P_k^{d}] = \frac{2^{-m}(m-1)!}{\Gamma(m)}(1-(v^2+1)^{-1/2})^m.\sum_{k=0}^{m-1}2^{-k}\binom{m-1+k}{k}(1+(v^2+1)^{-1/2})^k$$
$$v = \frac{\sigma}{\cos(\theta_e)}\sqrt{\frac{2m}{\Omega}} \quad (40)$$

Fig.3 shows the average probabilities of errors' comparisons in the cases of BPSK, the decorrelator detector, and the proposed detector for Nakagami flat fading channel of *m*=1, 2, &4.

4.3 Nakagami Multipath Fading Model

In the case of multipath channel model, the DS-CDMA received signal is represented as in Eq.(41).

$$r(t) = \sum_{k=1}^{K}\sum_{i=1}^{L} b_k A_{ki} s_k(t) + \sigma\, n(t) \quad (41)$$

To detect the desired user signal, L-fingers rack receiver is used with MRC (Maximum Ratio Combiner). In each finger the proposed detector is used to eliminate the multiple access interference. After using the proposed detector, a MRC is used to compensate the multipath fading effect. The output of the MRC is represented in Eq.(42).

$$DS_k = \sum_{i=1}^{L} \left|A_{ki}\right|^2 (1-\rho_{kr})\cos(\theta_{ie}) b_k$$
$$+\sigma \sum_{i=1}^{L} \left|A_{ki}\right| (n_{ki}\cos(\theta_{nk}(i)) - n_{ri}\cos(\theta_{rk}(i))) \qquad (42)$$
$$n_{ki} = e^{-j\theta_i}\int_0^T n(t).s_k^{*}(t).dt \quad , \quad n_{ri} = e^{-j\theta_i}\int_0^T n(t).s_r^{*}(t).dt$$

By following the same assumptions of θ_{ie}, $\theta_{nk}(i)$, and $\theta_{rk}(i)$ as in flat fading case, the mean and the variance values of the decision statistic in equation (42) can be represented as in Eqs.(43-44).

$$E[DS_k] = \sum_{i=1}^{L} \left|A_{ki}\right|^2 \cos(\theta_{ie})(1-\rho_{kr}) \qquad (43)$$
$$E[DS_k^2 - E[DS_k]^2] = 2\sigma^2 \sum_{i=1}^{L} \left|A_{ki}\right|^2 \qquad (44)$$

So, the probability of error can be represented as:

$$P_k^{pd} = \frac{1}{2} erfc\left(\frac{(1-\rho_{kr}).\sqrt{\sum_{i=1}^{L} \left|A_{ki}\right|^2 \cos^2(\theta_{ie})}}{2\sigma} \right) \qquad (45)$$

The random variable $|A_{ki}|$ is assumed to have a Nakagami *m*-distribution. The probability density function of $|A_{ki}|$ is given in Eq.(36). Also, $|A_{ki}|$ can be considered as a Rayleigh random variable with $2m$ degree of freedom. So, $|A_{ki}|^2$ is a chi-square random variable with $2m$ degree of freedom too. If $R_i = |A_{ki}|^2\cos^2(\theta_{ik})$, the probability density function and characteristic function of R_i is given as in Eq.(46) and Eq.(47) respectively.

$$f_{R_i}(R_i) = \frac{1}{\Gamma(m_i)}\left(\frac{m_i}{\Omega_i \cos^2(\theta_{ie})}\right)^{m_i} .R_i^{m_i-1}.e^{-\frac{m_i}{\Omega_i\cos^2(\theta_{ie})}R_i} \qquad (46)$$
$$\Phi_{R_i}(S) = \left(\frac{m_i/\Omega_i\cos^2(\theta_{ie})}{S + m_i/\Omega_i\cos^2(\theta_{ie})}\right)^{m_i} \qquad (47)$$

By assuming that the random variables R_i (i=1→L) are statistically independent. So, the random variable $R = \sum_{i=1}^{L} R_i$ has a characteristic function equaled to:

$$\Phi_R(S) = \prod_{i=1}^{L}\left(\frac{m_i/\Omega_i\cos^2(\theta_{ie})}{S + m_i/\Omega_i\cos^2(\theta_{ie})}\right)^{m_i} \qquad (48)$$

The probability density function of R is represented by:

$$f_R(R) = \frac{1}{\Gamma(m_t)}\left(\frac{m_t}{\Omega_t}\right)^{m_t} R^{m_t-1} e^{-\frac{m_t}{\Omega_t}R} \qquad (49)$$
$$m_t = \sum_{i=1}^{L} m_i \qquad and \qquad \Omega_t = \sum_{i=1}^{L}\Omega_i\cos^2(\theta_{ie})$$

From Eq.(45) and Eq.(49), the average probability of error for the proposed detector in Nakagami multipath channel in given by:

$$E[P_k^{pd}] = \int_{-\infty}^{\infty} P_k.f_R(R).dR$$
$$= 2^{-m_t}\left(1-(q^2+1)^{-\frac{1}{2}}\right)^{m_t} \sum_{i=1}^{m_t-1} 2^{-i}\binom{m_t-1+i}{i}\left(1+(q^2+1)^{-\frac{1}{2}}\right)^{i} \qquad (50)$$
$$q = \frac{\sqrt{2}\sigma}{(1-\rho_{kr})}\sqrt{\frac{2m_t}{\Omega_t}}$$

For the decorrelator detector, the procedure of average probability of error calculations is followed. The resulted formula of decorrelator average probability of error in Nakagami multipath fading channel is given in Eq.(51).

$$E[P_k^{d}] = \int_{-\infty}^{\infty} P_k.f_R(R).dR$$
$$= 2^{-m_t}\left(1-(q^2+1)^{-\frac{1}{2}}\right)^{m_t} \sum_{i=1}^{m_t-1} 2^{-i}\binom{m_t-1+i}{i}\left(1+(q^2+1)^{-\frac{1}{2}}\right)^{i} \qquad (51)$$
$$q = \frac{\sigma}{\sqrt{\mathbf{1-a_k^T R_k^{-1} a_k}}}\sqrt{\frac{2m_t}{\Omega_t}}$$

By the same way, the average probability of error for BPSK in multipath Nakagami fading channel is represented in Eq.(52).

$$E[P_k^{BPSK}] = \int_{-\infty}^{\infty} P_k.f_R(R).dR$$
$$= 2^{-m_t}\left(1-\left(\frac{2m_t\sigma^2}{\Omega_t}+1\right)^{-\frac{1}{2}}\right)^{m_t} \sum_{i=1}^{m_t-1} 2^{-i}\binom{m_t-1+i}{i}\left(1+\left(\frac{2m_t\sigma^2}{\Omega_t}+1\right)^{-\frac{1}{2}}\right)^{i} \qquad (52)$$

Figs.(4-6) show the average probability of error for the decorrelator detector, the proposed detector, and the BPSK receiver in Nakagami multipath channel with *m*=1, 2, &4 and L=1, 2, &4.

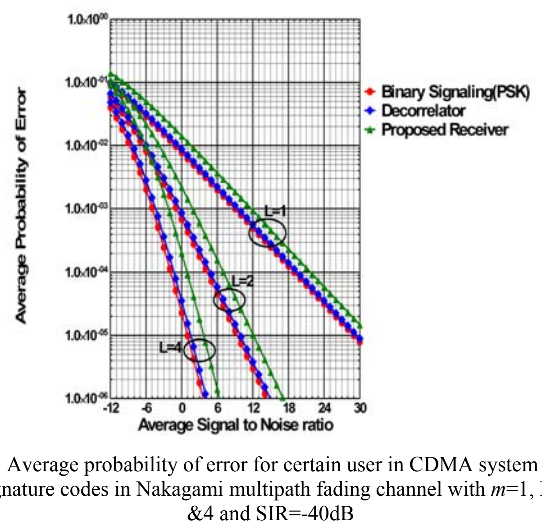

Fig.4. Average probability of error for certain user in CDMA system using ML signature codes in Nakagami multipath fading channel with *m*=1, L=1, 2, &4 and SIR=-40dB

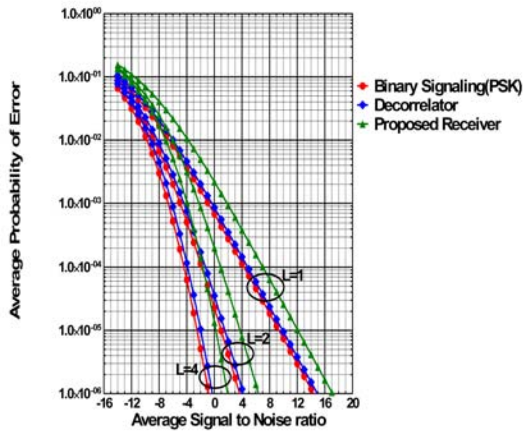

Fig.5. Average probability of error for certain user in CDMA system using ML signature codes in Nakagami multipath fading channel with *m*=2, L=1, 2, &4 and SIR=-40dB

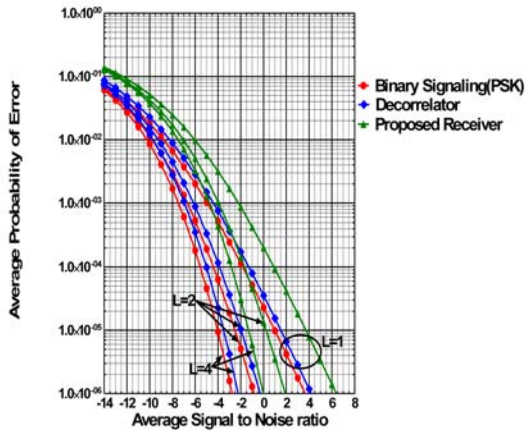

Fig.6. Average probability of error for certain user in CDMA system using ML signature codes in Nakagami multipath fading channel with *m*=4, L=1, 2, &4 and SIR=-40dB

5 SIMULATION RESULTS

In this section, simulation comparisons among the matched filter detector, the decorrelator detector, and the proposed detector are shown using three different criterions.

i. The average bit error rate criterion.
ii. The interference power measurement at the detectors' outputs given that the signal to interference ratio at the detectors' inputs are adjusted to two fixed different values in two different cases (-20dB and -40 dB).
iii. Detector noise gain criterion. This criterion is defined as the ratio between the noise power at the detector output and the noise power at the detector output.

These three different criterions help in putting up a complete clear view on the performance of the proposed CDMA detector with respect to the performance of the decorrelator detector and matched filter detector as two different linear CDMA detectors.

The simulations are done using maximal length signature codes. The simulations are done at two different SIR values at detectors' inputs. These SIR values are chosen to be smaller than -13 dB. From the CDG (CDMA Development Group) testing standards, the SIR value of -13dB is the common reference value of interference at CDMA detector input in any CDMA network [13]. The average received SNR value at different detector's inputs is varied from (– 20 dB) to (40 dB). The BER curves are plotted verses the average received signal to noise ratio at certain signal to interference ratio.

To limit the simulation's figures, the figures of bit error rate for certain cases are shown only. Figs.(7-9) shows the bit error rate curves of the pre-mentioned standard multiuser CDMA detectors using a data packet of length $\mathbf{10^5}$ bits for 5-user CDMA system at different signal to noise ratios for Nakagami fading channel with L=1 and *m* = 1 & 2. The users' signature codes are maximal length codes of period **31**. The input SIR value is -**40** dB. The average input SNR is varied between **-20** dB to **40** dB steps **2** dB.

From figs.(7-9), it was clear that the bit error rate performance of the proposed detector is approximately 3dB or less worst than the performance of the decorrelator detector. The interference measurement criterion has shown that the output multiple access interference power from the proposed detector is zero as the output multiple access interference of the decorrelator detector. The detector noise gain has shown that the noise power at the proposed detector output is less than or equal to 3dB more than the noise power at the decorrelator detector output.

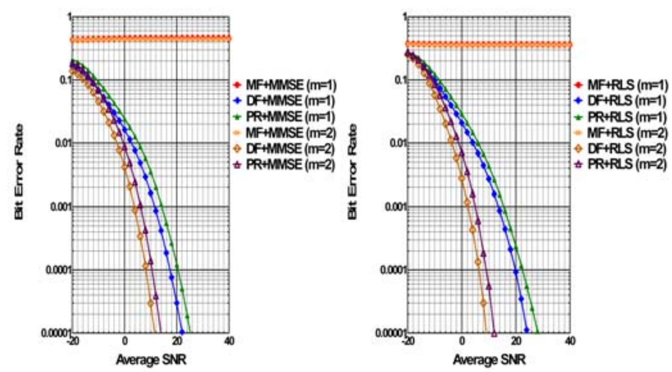

Fig.7. Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in Nakagami fading channel with L=1 and SIR=-40dB using MMSE and RLS channel estimation and FD=1 KHz

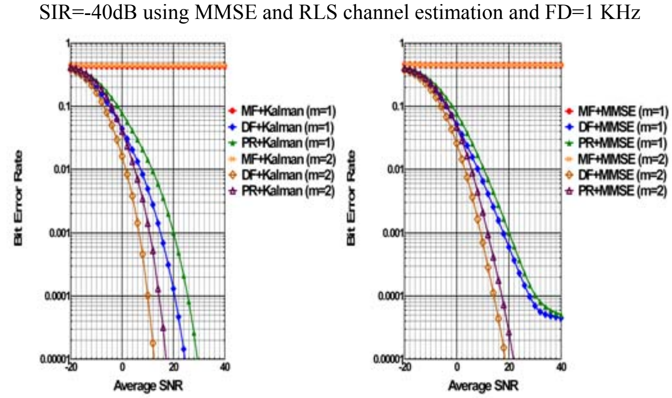

Fig.8. Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in Nakagami fading channel with L=1 and SIR=-40dB using Kalman filter and MMSE channel estimation and FD=1 KHz and 100K Hz

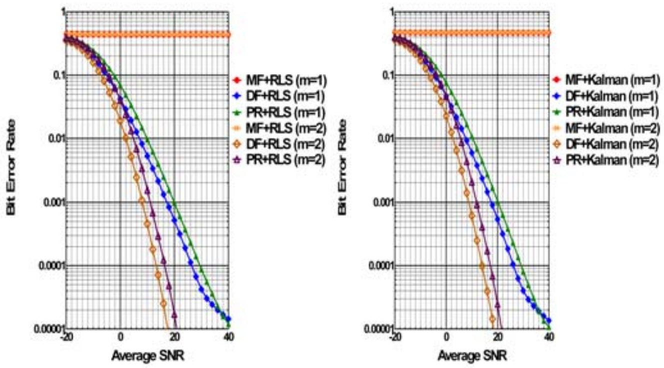

Fig.9. Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in Nakagami fading channel with L=1 and SIR=-40dB using RLS and Kalman filter channel estimation and FD=100K Hz

Two different Doppler frequencies are used to simulate slow and fast fading channels. Also, MMSE, RLS and Kalman filter algorithms are used to estimate the time varying channel's complex gains.

Also the bit error rate is better if the Nakagami fading shape factor (*m*) is increased. By comparing the simulation results with the mathematical results of Eqs.(38-40) and fig.(3), it was observed that the estimation phase error affects the performance of the detector. At slow fading, the MMSE estimation is better than the estimation of RLS and Kalman filter. On the other hand at fast fading the RLS and Kalman filter estimation is better than MMSE estimation. The used channel estimators can tack the channel amplitude better than the channel phase. As the Nakagami fading shape factor (*m*) increases, the error in estimating of the channel amplitude is smaller however the channel phase estimation error does not affected by this increasing in fading shape factor.

6 CONCLUSIONS AND FUTURE WORK

The Decorrelator detector needs to know all the users' signatures codes in CDMA system. Also, this type of detectors needs to calculate the correlation matrix among these users' codes and the inversion of this matrix. These requirements may not be easy to achieve. The new proposed detector that is based on two matched filters only does not need to know all signature codes of all system users nor calculation of correlation matrix and its inversion. This new proposed detector is as simple as the matched filter detector but has the same MAI cancellation of the decorrelator detector. The disadvantage of this new proposed detector is the duplication of system noise.

This new proposed linear detector is based on a mathematical observation relating to the symmetry property of the cross-correlation matrix among the CDMA system users' signature codes. This new proposed detector with simpler structure may help in increasing CDMA system capacity by allowing more number of system's users to share the same CDMA system's resources.

An expression for the probability of error for the proposed detector and the decorrelator detector are proposed for Nakagami flat and multipath channels. These expressions include the effect of phase estimation error on the probability of error. These expressions also help on the design of any CDMA receiver based on the proposed detector or the decorrelator detector in Nakagami fading channel by specifying the lower probability of the receiver that can be achieved by the used detector.

In the near future, it may be required to design a compete receiver based on adaptive detector such as MMSE or RLS with the proposed detector that can work in Nakagami fast fading channel.

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